Derivace základních funkcí

1. $(c)' = 0,\ c \in \mathbb{R}$ (konst.), $x \in \mathbb{R}$,
2. $(x^r)' = r \cdot x^{r-1},\ r \in \mathbb{R},\ x \in \mathbb{R}^+$,
3. $(\sin x)' = \cos x,\ x \in \mathbb{R}$,
4. $(\cos x)' = -\sin x,\ x \in \mathbb{R}$,
5. $(e^x)' = e^x,\ x \in \mathbb{R}$,
6. $(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x},\quad x \in \mathbb{R} \setminus \left\{\dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$,
7. $(\operatorname{cotg} x)' = -\dfrac{1}{\sin^2 x},\quad x \in \mathbb{R} \setminus \{k\pi,\ k \in \mathbb{Z}\}$,
8. $(\ln x)' = \dfrac{1}{x},\quad x \in \mathbb{R}^+$,
9. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}},\quad x \in (-1, 1)$,
10. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}},\quad x \in (-1, 1)$,
11. $(\operatorname{arctg} x)' = \dfrac{1}{x^2+1},\quad x \in \mathbb{R}$,
12. $(\operatorname{arccotg} x)' = -\dfrac{1}{x^2+1},\quad x \in \mathbb{R}$,
13. $(a^x)' = a^x \ln a,\ a > 0,\ a \neq 1,\ x \in \mathbb{R}$,
14. $(\log_a x)' = \dfrac{1}{x \ln a},\ a > 0,\ a \neq 1,\ x \in \mathbb{R}^+$.

součet, rozdíl

$$(f \pm g)' = f' \pm g'$$

derivace součinu

$$(fg)' = f'g + fg'$$

derivace podílu

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

derivace **složené funkce**

$$(f(g(x)))' = f'(g(x))g'(x)$$

Derivace $f(x)^{g(x)}$

Příklad

$$f(x) = x^{\sin x}, \quad x \in (0, \infty)$$

$$\ln f(x) = \sin x \ln x$$

$$\frac{1}{f(x)} f'(x) = (\sin x)' \ln x + \sin x \, (\ln x)'$$

$$f'(x) = \underbrace{x^{\sin x}}_{f(x)} \cdot \left(\cos x \ln x + \sin x \cdot \frac{1}{x} \right)$$

Příklad

$$f(x) = \sin x^{\cos x}, \quad x \in (0, \pi)$$

$$\ln f(x) = \cos x \ln \sin x$$

$$\frac{1}{f(x)} f'(x) = (\cos x)' \ln \sin x + \cos x \, (\ln \sin x)'$$

$$\frac{1}{f(x)} f' = \underbrace{\sin x^{\cos x}}_{f(x)} \cdot \left(-\sin x \ln \sin x + \cos x \cdot \frac{1}{\sin x} \cdot \cos x \right)$$