

Část A

Vypočtete integrál $\iiint_{\Omega} f(x, y, z) \, dx dy dz$ v kartézských souřadnicích.

Zadané funkce $f(x, y, z)$ a integrační oblasti Ω :

$$\Omega: x = 0, y = 0, z = 0, x + y = 2, x + y = z \quad f = x^2 + y^2 \quad (1)$$

$$\Omega: x = 0, y = 0, z = 0, 2x + 3y = 6, z = 2 \quad f = x^2 + y^2 + z^2 \quad (2)$$

$$\Omega: x = 0, y = 0, z = 0, x + z = 3, y = 4 \quad f = \frac{x+z}{4+y} \quad (3)$$

$$\Omega: x = 0, y = 0, z = 0, x + 2y + 3z = 6 \quad f = x \quad (4)$$

$$\Omega: x = 0, y = 0, z = 0, x + y + z = 1 \quad f = \frac{1}{1+x+y} \quad (5)$$

$$\Omega: x = 0, y = 0, z = 0, x + y + z = 1 \quad f = \frac{1}{(1+x+y+z)^2} \quad (6)$$

$$\Omega: x = 0, y = 0, z = 0, x + y + z = 1 \quad f = \frac{1}{(1+x+y+z)^3} \quad (7)$$

$$\Omega: x = 0, y = 0, z = 0, x + y + z = 1 \quad f = \frac{1}{(1+z)^3} \quad (8)$$

$$\Omega: 0 \leq x \leq \pi, 0 \leq y \leq \frac{\pi}{2}, 0 \leq z \leq \sqrt{\sin x \sin y} \quad f = z \sin x \quad (9)$$

$$\Omega: 0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq xy \quad f = x^3 y^2 z \quad (10)$$

Část B

Vypočtete integrál $\iiint_{\mathcal{M}} f(x, y, z) \, dx dy dz$ v (zobecněných) cylindrických souřadnicích.

Zadané funkce $f(x, y, z)$ a integrační oblasti \mathcal{M} :

$$\mathcal{M}: 0 \leq x \leq 1, 0 \leq y \leq \sqrt{1-x^2}, 0 \leq z \leq \sqrt{1-x^2-y^2} \quad f = xz \quad (11)$$

$$\mathcal{M}: 0 \leq x \leq 1, 0 \leq y \leq \sqrt{1-x^2}, 0 \leq z \leq \sqrt{1-x^2-y^2} \quad f = z(x^2 + y^2) \quad (12)$$

$$\mathcal{M}: 0 \leq x \leq 1, 0 \leq y \leq \sqrt{1-x^2}, \sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2} \quad f = xz \quad (13)$$

$$\mathcal{M}: z = 3x^2 + 3y^2, z = 1 - x^2 - y^2 \quad f = 1 \quad (14)$$

$$\mathcal{M}: x^2 + y^2 \leq 2x, y \geq 0, 0 \leq z \leq 3 \quad f = z\sqrt{x^2 + y^2} \quad (15)$$

$$\mathcal{M}: 4x^2 + y^2 \leq 16, x \geq 0, y \geq 2x, 0 \leq z \leq 3 \quad f = 2x + y - z \quad (16)$$

$$\mathcal{M}: 4 \leq x^2 + y^2 \leq 16, 0 \leq z \leq 1 \quad f = 1 \quad (17)$$

$$\mathcal{M}: 1 \leq x^2 + y^2 \leq 9, y \leq x, y \leq -x, 0 \leq z \leq 3 \quad f = x + y \quad (18)$$

$$\mathcal{M}: x^2 + y^2 \leq 3, x \leq y \leq x\sqrt{3}, 0 \leq z \leq 4 \quad f = z \quad (19)$$