ℓ'Hospitalovo pravidlo

$x_0 \in \mathbb{R}^*, \quad \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ nebo $\dfrac{f(x)}{g(x)} = \dfrac{\pm\infty}{\pm\infty}$ $\quad \lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to x_0} \dfrac{f'(x)}{g'(x)},$
pokud pravá limita existuje.

$\dfrac{0}{0}: \lim\limits_{x \to 0} \dfrac{\operatorname{arctg} x}{x} = \dfrac{1}{1+1^2} = \dfrac{1}{2}$ $\qquad \dfrac{\infty}{\infty}: \lim\limits_{x \to \infty} \dfrac{\ln x}{x} = \dfrac{1}{\infty} = 0$

- $0 \cdot \infty$: $\lim\limits_{x \to x_0} f(x) \cdot g(x)$ upravíme: $\lim\limits_{x \to x_0} \dfrac{f(x)}{\frac{1}{g(x)}}$ nebo $\lim\limits_{x \to x_0} \dfrac{g(x)}{\frac{1}{f(x)}}$

  $\lim\limits_{x \to 0} x e^{\frac{1}{x}} = \lim\limits_{x \to 0} \dfrac{e^{\frac{1}{x}}}{\frac{1}{x}} = \infty$ $\qquad \lim\limits_{x \to 0} (1-x^2) \operatorname{tg} \dfrac{\pi x}{2} = \dfrac{1-x^2}{\operatorname{cotg} \frac{\pi x}{2}} = \dfrac{4}{\pi}$

- $\infty - \infty$ upravíme např. $\lim\limits_{x \to x_0} f(x) - g(x) = \lim\limits_{x \to x_0} f(x) \left(1 - \dfrac{g(x)}{f(x)}\right)$

  nebo převedeme na společný jmenovatel.

  $$\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{1}{\ln x}\right) = \lim_{x \to 1} \frac{\ln x - (x-1)}{(x-1)\ln x} = \lim_{x \to 1} \frac{\frac{1}{x} - 1}{\ln x + \frac{x-1}{x}} =$$
  $$\lim_{x \to 1} \frac{-\frac{1}{x^2}}{\frac{1}{x} + \frac{1}{x^2}} = -\frac{1}{2}$$

Jiné nedefinované výrazy

- $1^\infty, \quad 0^\infty, \quad 0^0$:
$$f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x)\ln f(x)}, \qquad f(x) > 0$$
$$\lim_{x\to x_0} f(x)^{g(x)} = L \Rightarrow \lim_{x\to x_0} g(x)\ln f(x) = \ln L$$
- $\lim\limits_{x\to\infty} \left(1+\frac{1}{x}\right)^x = L$

$$\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \ln L$$

$$\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{1}{\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = 1$$

$$\ln L = 1 \quad \Rightarrow \quad L = e \quad \Rightarrow \quad \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$$